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January 3, 1958

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Dear Sir:

This letter report summarizes the effort under Task Order No. P during the period May 31 through December 31, 1957. The objective of this activity was the preparation of a gas-metering handbook, based on hydrogen as the working fluid, for use in connection with balloon inflation. The principal function of the handbook was to provide tables of lift at various pressures and temperatures; pressure, in atü (atmospheres, gage), ranged from 0.5 to 200 atü, and temperature from -40 to +40 C (-40 to +104 F).

Development of Lift Equation

As a first step, it was necessary to derive an equation for determining lift, so that a program for the 650 Computer could be written. The derivation may be summarized as follows:

The working fluid was assumed to be hydrogen with a small amount of nitrogen as an impurity. The gas mixture for which calculations were made was 98.5 mole per cent hydrogen and 1.5 mole per cent nitrogen.

The total available lift is given by the volume displaced, multiplied by the density difference for the gases involved, or:

$$\text{Lift} = (d_a - d_m)V_d, \tag{1}$$

where d_a = density of the air displaced

d_m = density of the hydrogen-nitrogen mixture

V_d = volume of gas mixture delivered.

The densities were calculated from the ideal gas law, with a compressibility factor included to compensate for deviations from ideality (PV = znRT):

$$d = \frac{n}{V}M = \frac{P}{zRT}M, \tag{2}$$

where P = pressure in atmospheres

R = gas constant in liter atm/°K-g-mole

T = absolute temperature in degrees Kelvin

z = compressibility factor

M = molecular weight

d = density.

Since the average molecular weight of a 98.5 per cent hydrogen-nitrogen mixture is 2.41, at atmospheric pressure:

$$d_m = \frac{0.02929}{T} \text{ kg/liter.} \tag{3}$$

The average molecular weight of the air displaced is given by:

$$M = \frac{1}{1 + H} \times 28.96 + \frac{H}{1 + H} \times 18.02, \tag{4}$$

where H = absolute humidity in moles of water per mole of dry air.

From Equation (2):

$$d_a = \frac{1}{zRT}\left(\frac{28.96 + 18.02\ H}{1 + H}\right). \qquad (5)$$

From saturation data*, a relation between H and the absolute temperature in degrees Kelvin was determined for a percentage humidity of 68:

$$H = 1.561 \times 10^{0.0257\ T\ -\ 9.518}. \qquad (6)$$

The volume of air displaced is equal to the volume of gas mixture delivered:

$$V_d = \frac{n_d RT}{zP_2}, \qquad (7)$$

where P_2 = balloon pressure

n_d = number of moles delivered to the balloon

Also, $n_d = n_1 - n_2$ = the initial number of moles in the cylinder minus the number of moles after evacuation.

$$n_1 = \frac{P_1 V_c}{zRT}, \qquad (8)$$

where P_1 = the initial pressure in atmospheres

V_c = cylinder internal volume in liters.

$$n_2 = \frac{PV_c}{zRT}, \qquad (9)$$

where P = 1.5 atmospheres (in order that the residual gas remain pure, it is expected

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*John H. Perry, "Chemical Engineers' Handbook", 1941, p 1082.

that the operator will leave a residual pressure of 0.5 atu").

z = approximately 1.001 over the temperature range involved.

Thus, we have:

$$n_d = n_1 - n_2 = \frac{V_c}{RT}\left(\frac{P_1}{z} - 1.4988\right) \text{ g-moles.} \qquad (10)$$

From Equation (7) we have:

$$V_d = \frac{n_d RT}{zP_2} = n_d RT = V_c\left(\frac{P_1}{z} - 1.4988\right) \text{ liters.} \qquad (11)$$

Substituting Equations (3), (5), (6), and (9) in Equation (1), combining constants, and simplifying, for the 6-cubic-meter cylinder we have:

$$\text{Lift} = \qquad (12)$$

$$\frac{8.789}{T}\left(\frac{P_1}{z} - 1.4988\right)\left(\frac{1.607 + 1.561 \times 10^{0.0257T - 9.518}}{1 + 1.561 \times 10^{0.0257T - 9.518}} - 0.1334\right).$$

If the factor of humidity had not been considered, an error of over 3 per cent in lift could result. By the assumption of 68 per cent humidity (an average value determined from conditions in major U. S. cities), the possible error was reduced to less than 1 per cent.

Correlation of Data and Determination
of Equation of State

In order to solve the derived lift equation at the desired conditions of temperature and pressure, an equation of state for the gaseous mixture was required. Since for the most part experimental P-V-T data are given in the form of compressibilities, it is most convenient to retain this means of expression. As seen earlier, the lift equation is written in terms of pressure, temperature, and compressibility only.

A literature search was undertaken to gather data on gaseous hydrogen and nitrogen. The data* obtained appeared as compressibility versus absolute pressure in atmospheres at various Kelvin temperatures. A relation between these variables was determined, with the aid of the 650 Computer, in the following manner:

For both gases, a relation of the form $z = A + BP + CP^2$ was assumed for each isotherm. The constants were evaluated by the method of least squares, giving a set of A, B, and C values at each temperature considered. In a similar manner, these values were related to the absolute temperature by least-squares fits of the form $A = a + bT + cT^2$ for each gas.

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*"Tables of Thermal Properties of Gases", U. S. Department of Commerce, N.B.S. Circular 564, pp 271-274, 317-323.

H. L. Johnston and D. White, "PVT Relationships of Gaseous Normal Hydrogen", Transactions ASME, August, 1950, pp 785-787.

Deming and Shupe, "Physical Review", 1931, p 638.

If Amagat's Law is assumed to apply, the mean compressibility factor of a mixture may be given by $z_m = x_1 z_1 + x_2 z_2$, where x is the mole fraction. It has been shown* that for mixtures of hydrogen and nitrogen, especially in the purity range considered, this assumption may be safely made. The corresponding constants, e.g., a_{H_2} and a_{N_2}, were thereby combined, resulting in a final relation of the form:

$$z_{H_2-N_2} = A' + B'P + C'P^2, \qquad (13)$$

$$\text{where } A' = a' + b'T + c'T^2$$
$$B' = d' + e'T + f'T^2$$
$$C' = g' + h'T + j'T^2.$$

Substituting the constants, as determined by machine computation, we have:

$$z = 0.990 + 7.09\text{x}10^{-5}T - 1.24\text{x}10^{-7}T^2 + (7.66\text{x}10^{-4} - 1.39\text{x}10^{-9}T)P + (3.24\text{x}10^{-6} - 1.74\text{x}10^{-8}T + 2.45\text{x}10^{-11}T^2)P^2. \qquad (14)$$

Substituting Equation (14) in Equation (12) results in an equation relating lift with temperature and pressure only.

General Description of Handbook

The inflation tables constituted the bulk of the handbook.

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*Hougen and Watson, "Industrial Chemical Calculations", 1936, p 404.

However, it was necessary to provide introductory information on the handling of hydrogen cylinders and the use of the handbook, together with related conversion charts or tables. The discussion and descriptive material, tables, and charts that were provided as introductory sections in the handbook are indicated below:

A. Discussion and descriptive material

(1) Description of hydrogen cylinders and controls commonly encountered

(2) Description of tables included in the handbook

(3) Discussion of safety measures

(4) Discussion of weight of flight equipment and free lift

(5) Description of recommended temperature-measurement procedure

(6) Description of recommended pressure-measurement procedure

(7) Discussion of conversion factors for cylinders other than the German 6-cubic-meter size on which the inflation tables were based

(8) Discussion of corrections for gas impurities other than 1.5 per cent nitrogen

(9) Discussion of partial-cylinder correction

(10) Sample problems illustrating the use of the handbook.

B. Tables

(1) Cylinder-conversion factors. This table lists a number of standard German and U. S. cylinders together with the conversion factors, determined from the following relationship:

$$\text{Conversion Factor} = \frac{\text{Lift Determined From Inflation Tables}}{\text{Actual Lift From Cylinder}}.$$

The reason that this simple conversion is possible can be found by examining Equations (1) and (9) above, which show that lift is a direct function of cylinder internal volume.

(2) Lift-conversion table. This table converts from pounds, force, to kilograms, force, and covers the range from 0.1 to 22.5 pounds in 0.1-pound increments.

(3) Pressure-conversion table. This table converts from atü (atmospheres, gage) to psig (pounds per square inch, gage), and covers the range from 0.5 to 200 atü in 0.5-atü increments.

C. Charts

(1) Correction charts for gas impurities. Correction charts are provided for impurities of nitrogen, oxygen, carbon dioxide, and ammonia. To simplify the use of the charts, the compensation for impurities is based on 15 C and is correct at only that temperature. The correction applicable at other temperatures will be in error to the extent that the compressibility factor varies with temperature. However, the compressibility factor varies with pressure to a much greater degree than with temperature, and for this reason pressure was used as a parameter on the correction charts. The correction is:

$$\text{Fractional Lift (Correction Factor)} = \frac{\text{Actual Lift}}{\text{Lift Determined From Inflation Tables}}.$$

(2) Partial-cylinder correction chart. If a cylinder is partially drained, expansion of the gas causes cooling, which reduces the pressure in the cylinder. If the cylinder is allowed to warm to ambient temperature, the pressure will increase. When the cylinder is again at equilibrium

with the ambient temperature, the inflation tables can be used to determine the amount of lift remaining in the cylinder. However, it is normally desirable to determine this value immediately after use of the cylinder. For this reason, the partial-cylinder correction chart has been provided to permit estimating the pressure to which a cylinder should be drained in order to yield the desired equilibrium pressure. The degree of cooling, and therefore the pressure loss for any cylinder, depends upon a number of factors, such as rate of discharge, and size and mass of the cylinder.

To develop this chart, a number of U. S. Linde Standard 191-cu-ft cylinders were drained and data taken as to the instantaneous and equilibrium pressures. Since these cylinders are nearly the same size as the German 6-cubic-meter cylinder (conversion factor for the Linde Standard cylinder is 0.949), the data provided by the chart are equally applicable to the German cylinder.

The gas-metering handbook developed in the course of this project will provide an accurate means of determining the degree of balloon inflation necessary to obtain a desired lift with a minimum of computation. The handbook was reproduced, and 250 copies will be sent to the Sponsor in the near future.

We would appreciate receiving any comments that you or your associates might care to make on the handbook prepared or the work done on this project.

Sincerely,

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In Triplicate